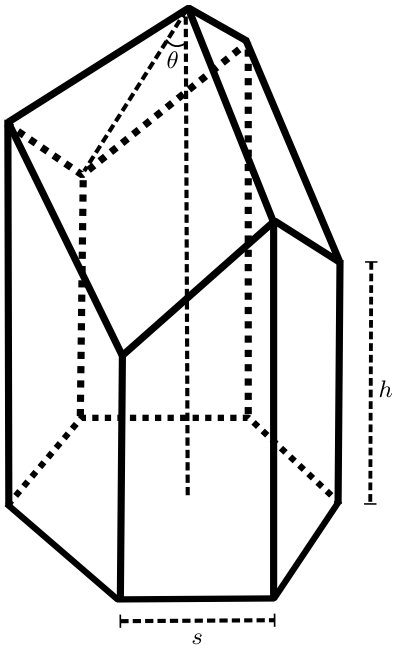
1. **Draining a Trough.** A trough in the shape of a half cylinder has length 5m and radius 1m. The trough is full of water when a valve is opened, and water flows out of the bottom of the trough at a rate of $1.5\text{m}^3/\text{hr}$. *Hint: The area of a sector of a circle of radius r subtended by an angle θ is $r^2\frac{\theta}{2}$.*

   (a) How fast is the water level changing when the water level is 0.5m from the bottom of the trough?

   (b) What is the rate of change of the surface area of the water when the water is 0.5m deep?

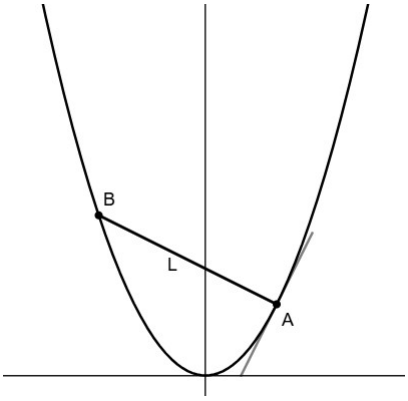
2. **Efficient as Bees.** Bees build honeycomb structures out of cells with a hexagonal base and three rhombus-shaped faces on tops as in Figure 20. Using geometry, we can show that the surface area of this cell is

$$A(\theta) = 6hs + \frac{3}{2}s^2(\sqrt{3}\csc\theta - \cot\theta).$$



Of course, bees know the angle that minimizes the surface area. So, find the value θ where $0 < \theta < \frac{\pi}{2}$ which minimizes $A(\theta)$ using EVT.

3. **Dancing on a Parabola.** Two people, A and B, walk along a parabola $y = x^2$ in such a way that the line segment L between them is always perpendicular to the line tangent to the parabola at A's position. Determine the positions of A and B when L has minimum length.



4. **Cauchy's Mean Value Theorem.** Suppose f and g are continuous on $[a, b]$ and differentiable on (a, b) where $g(a) \neq g(b)$. Show that there is a point c in (a, b) where

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}.$$